A pseudo-LISA convention for the Mock LISA Data Archive

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A first draft of pseudo-LISA conventions for the LISA orbits, for GW source objects, for the LISA TDI responses, and for the standard TDI combinations. Mostly pasted together from Refs. [2] and [6].

PACS numbers:

I. INTRODUCTION

II. LISA ORBITS

We define the orbits of the pseudo-LISA spacecraft as defined in the Appendix of Ref. [2] (and as used in the *LISA Simulator*). Namely, in the a Solar-system–baricentric ecliptic coordinate system (SSB frame) where we have set the x axis toward the vernal point, the coordinates of each spacecraft are given by the expressions

$$\begin{aligned} x &= r\left(\cos(\sqrt{3}e)\cos\beta\cos\gamma - \sin\beta\sin\gamma\right), \\ y &= r\left(\cos(\sqrt{3}e)\sin\beta\cos\gamma + \cos\beta\sin\gamma\right), \\ z &= -r\sin(\sqrt{3}e)\cos\gamma\,. \end{aligned} \tag{1}$$

where $\beta = 2(n-1)\pi/3 + \lambda$ ($n = 1, 2, 3$) is the relative orbital phase of each spacecraft in the constellation, γ is the ecliptic azimuthal angle, and r is the standard Keplerian radius

$$r = \frac{a(1-e^2)}{1+e\cos\gamma}\,. \tag{2}$$

Here a is the semi-major axis of the guiding center and has an approximate value of one AU. To get the above coordinates as a function of time we first note that the azimuthal angle is related to the eccentric anomaly, ψ, by

$$\tan\left(\frac{\gamma}{2}\right) = \sqrt{\frac{1+e}{1-e}}\tan\left(\frac{\psi}{2}\right), \tag{3}$$

and the eccentric anomaly is related to the orbital phase $\alpha(t) = 2\pi f_m t + \kappa$ through

$$\alpha - \beta = \psi - e\sin\psi\,. \tag{4}$$

For small eccentricities we can expand equations (3) and (4) in a power series in e to arrive at

$$\gamma = (\alpha-\beta) + 2e\sin(\alpha-\beta) + \frac{5}{2}e^2\sin(\alpha-\beta)\cos(\alpha-\beta) + \cdots \tag{5}$$

Substituting this series into equation (1) and keeping terms only up to order e gives us

$$\begin{aligned} x &= a\cos(\alpha) + a\,e\left(\sin\alpha\cos\alpha\sin\beta - (1+\sin^2\alpha)\cos\beta\right), \\ y &= a\sin(\alpha) + a\,e\left(\sin\alpha\cos\alpha\cos\beta - (1+\cos^2\alpha)\sin\beta\right), \\ z &= -\sqrt{3}\,a\,e\cos(\alpha-\beta)\,, \end{aligned} \tag{6}$$

with $e = 0.00965$, $a = 1$ AU. These are the desired coordinates of each spacecraft as a function of time. Notice that by keeping only linear terms in the eccentricity we are neglecting the variation in the optical path length. The path length will change due to the Keplerian orbits, but these effects enter at $\mathcal{O}(e^2)$ and above.[1]

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[1] However: the LISA Simulator and Synthetic LISA actually use expressions accurate to order e^2 for the positions. Synthetic LISA uses approximate armlengths accurate to order e. Update the above to reflect that?

These spacecraft orbits are mapped (exactly!) to those used in *Synthetic LISA* [4, 6] by setting the *Synthetic LISA* `EccentricInclined` parameters $\eta_0 = \kappa$, $\xi_0 = 3\pi/2 - \kappa + \lambda$, $sw < 0$ (which has the effect of exchanging spacecraft 2 and 3).

III. GRAVITATIONAL-WAVE SOURCES

We follow Ref. [2] (and the *LISA Simulator*) in describing the sky location of gravitational-wave sources by the unit vector $\hat{n}$,

$$\hat{n} = \sin\theta\cos\phi\,\hat{x} + \sin\theta\sin\phi\,\hat{y} + \cos\theta\,\hat{z}\,, \tag{7}$$

(where θ and ϕ are the J2000 *ecliptic colatitude* and *longitude*, the latter measured from the vernal point, aligned with the $\hat{x}$ axis in our convention). The corresponding gravitational radiation is modeled as a plane wave in a transverse-traceless gauge, propagating in the $\widehat{\Omega} = -\hat{n}$ direction in the SSB frame. The surfaces of constant phase are then given by $\xi = t + \hat{n}\cdot\mathbf{x} = \text{const}$. A generic gravitational wave can be decomposed into two standard polarization states,

$$\mathbf{h}(\xi,\hat{n}) = h_+(\xi)\mathbf{e}^+(\hat{u},\hat{v}) + h_\times(\xi)\mathbf{e}^\times(\hat{u},\hat{v})\,, \tag{8}$$

where $\mathbf{e}^+$ and $\mathbf{e}^\times$ are the polarization tensors

$$\begin{aligned} \mathbf{e}^+ &= \hat{u}\otimes\hat{u} - \hat{v}\otimes\hat{v}, \\ \mathbf{e}^\times &= \hat{u}\otimes\hat{v} + \hat{v}\otimes\hat{u}, \end{aligned} \tag{9}$$

and where

$$\begin{aligned} \hat{u} &= \cos\theta\cos\phi\,\hat{x} + \cos\theta\sin\phi\,\hat{y} - \sin\theta\,\hat{z}, \\ \hat{v} &= \sin\phi\,\hat{x} - \cos\phi\,\hat{y}\,. \end{aligned} \tag{10}$$

If we refer gravitational-wave emission to the *principal polarization axes* $\hat{p}$ and $\hat{q}$ of the source,

$$\mathbf{h}(\xi,\hat{n}) = h^S_+(\xi)\,\boldsymbol{\epsilon}^+(\hat{p},\hat{q}) + h^S_\times(\xi)\,\boldsymbol{\epsilon}^\times(\hat{p},\hat{q}), \tag{11}$$

with

$$\begin{aligned} \boldsymbol{\epsilon}^+ &= \hat{p}\otimes\hat{p} - \hat{q}\otimes\hat{q}, \\ \boldsymbol{\epsilon}^\times &= \hat{p}\otimes\hat{q} + \hat{q}\otimes\hat{p}\,. \end{aligned} \tag{12}$$

we can go back to the general decomposition (8) by setting

$$h_+(\xi) = \cos(2\psi)\,h^S_+(\xi) + \sin(2\psi)\,h^S_\times(\xi), \tag{13}$$

$$h_\times(\xi) = \cos(2\psi)\,h^S_\times(\xi) - \sin(2\psi)\,h^S_+(\xi), \tag{14}$$

where $\psi = -\arctan(\hat{v}\cdot\mathbf{p}/\hat{u}\cdot\mathbf{p})$ is the *source polarization angle*.

These conventions are mapped (exactly!) to those used in *Synthetic LISA* [4, 6] by setting the *Synthetic LISA* `Wave` parameters $\beta = \pi/2 - \theta$ (β is the J2000 *ecliptic latitude*), $\lambda = \phi$, and $\psi_{\mathrm{SL}} = -\psi_{\mathrm{LS}}$.

IV. LISA RESPONSES

The basic LISA response to gravitational waves is taken to be the *phase response* Φ_{ij} used in the *LISA Simulator* and discussed in Sec. II of Ref. [2] [see especially Eqs. (4)–(13) and (22)] or equivalently the *fractional frequency response* y^{gw}_{slr} used in *Synthetic LISA* and discussed in Sec. II B of Ref. [6] (i and s identify the transmitting spacecraft, j and r the receiving spacecraft for each phase measurement, l is a redundant link index).

The phase and fractional frequency formalisms are equivalent, and related by a simple time integration. It is not clear at this time which will be the primary format for LISA data, and perhaps both should be adopted concurrently. The frequency measurements have the advantage of being directly proportional to the gravitational strain; the phase measurements have the advantage of representing more closely the actual output of the LISA phasemeters.

V. TDI OBSERVABLES

We define the standard TDI observables following the *Synthetic LISA* [4, 6] naming scheme and sign conventions (see also the *Synthetic LISA* file `lisasim-tdi.cpp`). All of these can be used both as frequency and phase observables by replacing y_{slr} measurements with Φ_{ij} measurements. See the TDI Rosetta Stone [5] for translations between index notations (in particular, the primed indices of Ref. [8] correspond to positive indices in the *Synthetic LISA* usage).

- First-generation TDI (TDI 1.0): the *Sagnac* observables α, β, γ ("centered", respectively, on spacecraft 1, 2, 3, as all following sets of three), and the *symmetrized Sagnac* observable ζ, as defined in Ref. [7]. No need to define the eight-pulse observables (Michelson, etc.), which are the same as in modified TDI.

- Modified TDI (TDI 1.5): the *unequal-arm Michelson* observables X, Y, Z; the *relay* observables U, V, W; the *monitor* observables E, F, G; the *beacon* observables P, Q, R; the *Sagnac* observables α_1, α_2, α_3; and the *symmetrized Sagnac* observables ζ_1, ζ_2, ζ_3 as defined in Ref. [8].

- Second-generation TDI (TDI 2.0): the *unequal-arm Michelson* observables X_1, X_2, X_3; the *relay* observables U_1, U_2, U_3; the *monitor* observables E_1, E_2, E_3; the *beacon* observables P_1, P_2, P_3 as defined in Ref. [8].

- Optimal TDI observables: in first-generation TDI, A, E, and T as defined in terms of α, β, γ in Ref. [9]; in second-generation TDI, $\bar{A}$, $\bar{E}$, $\bar{T}$ as defined in terms of α_1, α_2, α_3 in Ref. [10].

Probably the modified and second-generation TDI Michelson observables will be those used most often for the MLDA. Note also that there is a naming conflict here between the first-generation spacecraft-1-centered monitor observable and the first-generation spacecraft-2-centered optimal observable.

VI. CONCLUSIONS

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